Probability Distributional Analysis of Hourly Solar Irradiation in Kumasi-Ghana

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ABSTRACT

The probability distribution of hourly solar irradiation for Kumasi –Ghana was conducted using 14 years of data of measured values by the KNUST SOLAR LABORATORY. The analysis was carried out to find out the probability distributions that best fit the data of a given month of the year. The analysis conducted, revealed that solar irradiation for January, March and May can be fitted with the lognormal probability distribution. The month of April can be fitted with the Exponential, Weibull, lognormal Geometric and Gamma distribution while the months of June to December can be fitted with Exponential, Weibull, Geometric and Gamma. The paper finally concludes that the solar irradiation distribution in Ghana is not normally distributed with the above mentioned distribution describing the respective month of the year.

Keywords: Solar irradiation, Probability distribution, Mean Square Error.

1.0 INTRODUCTION

Knowledge of the solar irradiation climate of an area is of paramount importance in assessing the potential use of a solar energy system, converted into either thermal or electrical energy, as a power source in that area. Such information is a prerequisite for the design of such solar energy conversion system.

The mobilization of adequate national financial resources for the planning and development of the local solar energy resource depends on the availability of solar radiation data which could be used to evaluate available resources and to assess the probable long term performance of systems and hence their economic viability. The solar radiation data should be measured continuously and accurately over the long term for the purposes of feasibility studies and building of solar powered systems. Unfortunately, in most areas of the world, solar radiation measurements are not easily available due to financial, technical and institutional limitations (Zaharim *et al*, 2009)

The solar radiation received at the earth's surface is subject to daily, seasonal and annual variations and hence many years of observation (perhaps at least 20 years) must be acquired in order to obtain a fairly accurate estimate of long term availability and distribution. However many locations in the developing countries do not have the facilities for continuous and accurate measurements of solar radiation and it is then necessary to use empirical methods which are based on easily measured meteorological parameters such as temperature, relative humidity, rainfall, cloudiness and duration of bright sunshine.

Many such formulae have been documented in the literature (Knight et al, 1991) although the most widely used correlation and perhaps the simplest, is the Angstrom (B) linear regression equation as modified by Page*etal* (1964). This correlation relates the monthly average, daily global irradiation on the horizontal to the relative duration of sunshine, and it has been applied to a variety of climates including tropical locations. Except for the recent work of Neba-Fabs et al., (1988) and of Exel(1978) nearly all the work done for locations in the West Africa sub-region and other tropical locations have sought to determine a single regression equation which could be used for all months and hence all seasons of the year.

There was however an indication that the Angstom-Page correlation coefficients depend on both the local climate and the season. Furthermore, it is anticipated that more accurate estimates of monthly average global irradiation would be obtained from correlations for particular months Eze *et al* (1988).

The extensive statistical analysis conducted on the daily global irradiation on the horizontal particularly investigating possible variations of the frequency distribution with both location and season. Their results showed that frequency distribution of daily global irradiation on the horizontal for the monthly period corresponding to a specified value of a monthly mean clearness index, is almost independent of the location and the time of the year. Liu and Jordan (1963) as well as Bendt et al (1981).They moreover, went ahead and showed that the generalized cumulative distribution function may be obtained from a probability density function which assumed among others random daily insolation sequences Bendt *et al*.

In this study, we shall not dwell on the regression methods used to estimate the monthly global averages which, in any case, has already comprehensively been dealt with by Jackson et al (1990) but rather, taking advantage of the currently abundant data on solar irradiation data for Kumasi, we undertake the determination of the pertinent probability density curves based on randomly selected samples in respect of monthly or seasonal variations.

1.2 MATERIALS AND METHODS

Pyranometer is the device used to record the sunlight data. It is made up of solar cell modules which harvest energy from the sun. The output of the solar cell modules depends on the amount of sunlight (or solar radiation) falling on them and it is affected by seasonal and daily solar radiation changes. It also changes depending on how cloudy or dusty the site is. It records two types of radiations: global average and diffused average.

The global average radiation is the hourly average irradiance of the direct solar energy reaching the earth's surface and the diffused average radiation is as a result of the direct solar energy being blocked by a black ring, clouds or dust before reaching the solar cell modules. However, this research work makes use of the global average radiation which is useful in the production of solar energy.

Data on hourly solar irradiation in Kumasi was collected from the Solar Energy Laboratory of the Mechanical Engineering department of KNUST, Kumasi. The irradiation data which was measured in kilowatt hours per meter squared was collected by means of a pyranometer. The data consists of fourteen years of hourly solar irradiation data from 1995-2008.

Descriptive statistical analyses as well as inferential statistical analysis were deployed to obtain the desired results. Probability curve fitting was applied to find the best fitted probability distribution for the various months of the year. The concept of the mean squared error (MSE) was applied to find estimate the best fitted probability distribution. The analysis was conducted on the hourly solar irradiation data obtained in the past years to gain much insight of the data to constructively solve the problems as stated.

1.3 STANDARD PROBABILITY DISTRIBUTIONS USED

1.3.1 The Exponential Distribution

The exponential probability distribution for the continuous random variable, X which represents an interval of time or space is defined as:

$$f\left(X\right)=\begin{cases}\theta e^{-\theta x} & \text{x>0}\\ 0 & \text{otherwise}\end{cases}.$$

Where, the parameter θ > 0 is the mean number of events that occur in the given unit of time or space. The mean value of X (that is, the mean length of time or space between successive occurrences of the event and the variance are:

$$E\left(X\right)=\frac{1}{\theta}\,Var\left(X\right)=\frac{1}{\theta^2}$$

1.3.2 The Normal Distribution.

The probability density function for the normal random variable, X which is simply called normal distribution is defined by:

$$f(X)=\begin{cases}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{(X-\mu)}{\sigma^2}\right)}, -\infty \le X \le \infty \\ 0 \qquad \text{otherwise}\end{cases}$$

σ > 0, - ∞ < μ < ∞ and the mean and variance of the measurements, x are;

$E(X)$ = μ and $Var(X)$ = σ^2

If the random variable is modeled by the Normal distribution with mean μ and variance, σ^2 then it is simply denoted as x ⊔ N (μ, σ^2). The graph of a normal distribution is a bell-shaped smooth curve.

The Normal distribution is one of the most widely used probability distributions for modeling random experiments. It provides a good model for continuous random variables involving measurements such as time, heights/ weight of persons, marks scored in an examination, amount of rainfall, growth rate and many other scientific measurements.

1.3.3 The Weibull Distribution

The continuous random variable X has a Weibull distribution, with parameters α and β if its density function is given by

$$f(X;\alpha,\beta)=\begin{cases}\alpha\beta X^{\beta-1}e^{-\alpha X^{\beta}} \\ 0, elsewhere\end{cases}, X>0$$

where α >0 and β >0.

The mean and variance of the Weibull distribution are given by

$$\mu=\alpha^{-1/\beta}\Gamma\left(1+\frac{1}{\beta}\right) \text{ and } \sigma^2=\left\{\Gamma\left(1+\frac{2}{\beta}\right)-\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\}$$

Modern technology has enabled us to design many complicated systems whose operation, or perhaps safety, depends on the reliability of various components. The Weibull distribution has been used extensively in recent times to deal with such problems. This is applied to reliability and life-testing problems such as time to failure or life length of a component, measured from specific time until it fails.

1.3.4 The Gamma Distribution

The continuous random variable X has a gamma distribution, with parameter α and β, if its density function is given by

$$f(X;\alpha,\beta)=\begin{cases}\frac{1}{\beta^{\alpha}\Gamma(\alpha)}X^{\alpha-1}e^{-X/\beta} \\ 0, elsewhere\end{cases}, X>0$$

Where, α >0 and β >0.

The gamma distribution for which α =1 is the exponential distribution. The mean and variance of the gamma distribution are

$\mu=\alpha\beta$ and $\sigma^2=\alpha\beta^2$

The exponential thus turns out to be a special case of the gamma distribution. The exponential gamma distribution plays an important role in both queuing theory and reliability problems. Time between arrivals at service facilities, and time to failure of component parts and electrical systems, often is nicely modeled by the exponential distribution. The relationship between the gamma and exponential allows the gamma to be involved in similar types of problems

1.3.5 Lognormal Distribution

The continuous random variable X has a lognormal distribution if the random variable $Y=\ln(X)$ has normal distribution with mean μ and standard deviation σ. The resulting density function of X is

$$f(X;\mu,\sigma)=\begin{cases}\dfrac{1}{\sqrt{2\Pi}\sigma X}e^{-\frac{1}{2\sigma^2}\left[\ln(X)-\mu\right]^2},x\geq 0\\ 0, elsewhere, X<0\end{cases}$$

The mean and variance of the lognormal distribution are

$$\mu=e^{\mu+\frac{\sigma^2}{2}}\text{ and }\sigma^2=e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$$

The lognormal distribution is used for a wide variety of applications. The distribution applies is cases where a natural log transformation results in a normal distribution. The cumulative distribution function is quite simple due to its relationship to the normal distribution.

1.3.6 Beta Distribution

The family of distribution most commonly used to model researchers' uncertainty about the unknown probability ***p***of some event is the family of beta distributions, defined as follows**:** Arandom variable X is said to have a beta distribution with parameters $\alpha>0\ \ \beta>0$ if the density of X is

$$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}\text{ for }0\leq x\leq 1\text{ and }f(x)=0\text{ outside the interval }[0,1]$$

The mean and the variance of the beta distribution are given below

$$Mean\text{ denoted }E(X)=\mu=\frac{\alpha}{\alpha+\beta}\text{ and variance denoted }\mathrm{Var}(X)=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

1.4 DATA ANALYSIS AND RESULTS

The analysis adopted only descriptive statistical method. This includes frequency and relative frequency distribution tables, probability density curves, mean square error and histograms.

Frequency Distribution

Since the sampled data were too many to gain an insight into, and impossible for one to convey much information about its characteristics, it became necessary to organize and reduce it into meaningful forms as follows:

"The data for each month was organized into groups, called classes or categories using the Sturges (approximation)."

1.4.1 Probability Distributions

To make inferences with respect to the population distribution of each month, some standard continuous probability distributions were selected based on their shape parameters to fit the data for each month. The distributions are the exponential, Weibull, Gamma, Lognormal and Beta, Least square and maximum likelihood estimation procedures were then applied to estimate the parameter for each distribution from the data. The probability density function and their respective histogram for the various months are shown below.

1.4.2 Mean Squared Errors

The mean squared error of the monthly data and each distribution selected were computed to come up with the best candidate fitting the data as follows: $MSE = \frac{\sum_{t=1}^{n}(rf_i - f(x_i))^2}{n}$

Where rf_i = each relative frequency for the month

$f(x_i)$ = values computed using the probability distribution under consideration.

x_i = the class mark(x), i.e. the hourly solar irradiance for the month under consideration.

1.5 PRESENTATION OF RESULTS

1.5.1 Histogram and Probability Density Function

A random sample of size one thousand five hundred was selected from each month of the year and probability density function and histogram in respect of hourly solar irradiance for the various months of the year was produced.

1.5.2 Probability Density Function and Histogram January-December

The histogram and the probability density function for the month of January are presented in the Fig 1.1 and Fig 1.1a below respectively.

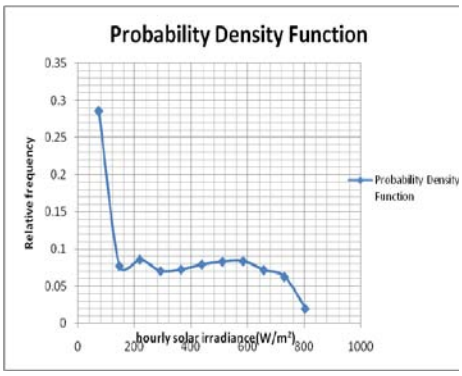


Fig 1.1: Probabilility density function for Jan

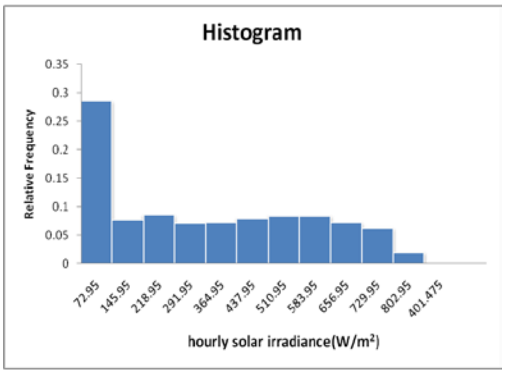


Fig 1.1a: Histogram for the month of Jan

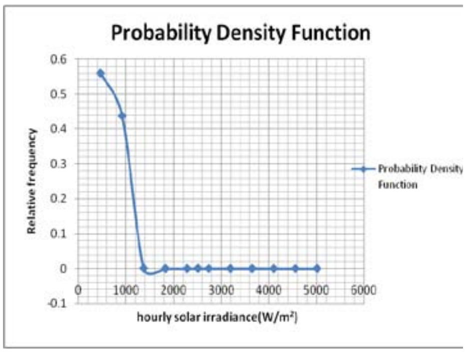


Fig 1.2:Probabilility density function for Feb

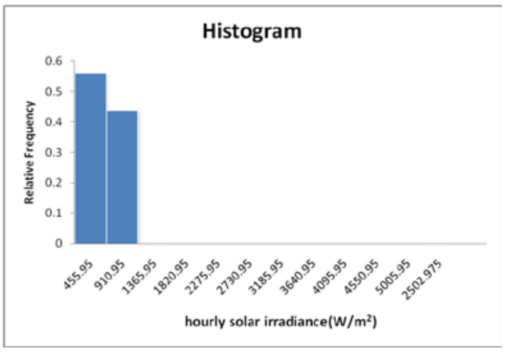


Fig 1.2a:Histogram for the month of Feb

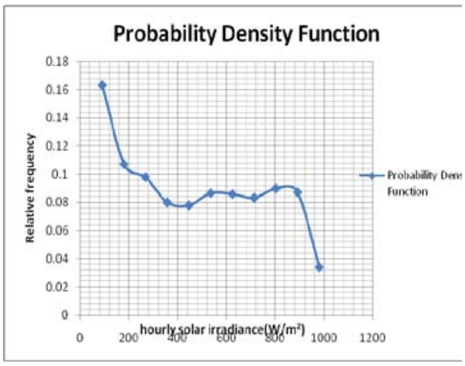


Fig1.3: Probability density function for March

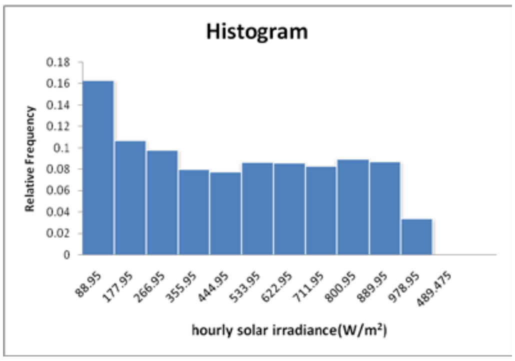


Fig 1.3a: Histogram for the month of March

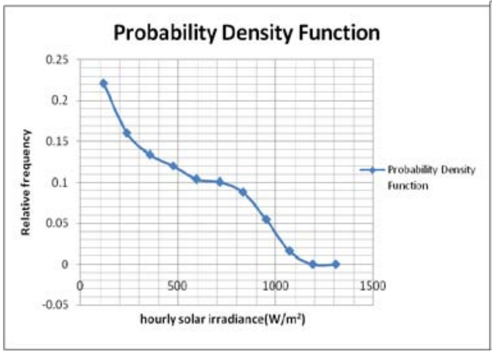


Fig 1.4: Probability density function for April

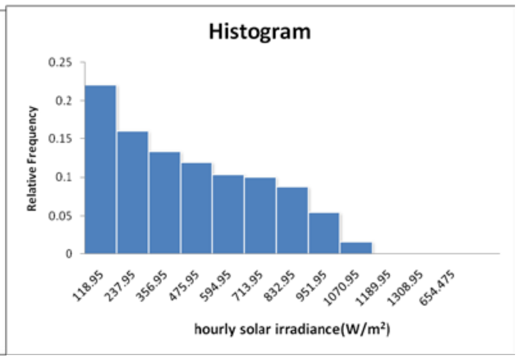


Fig 1.4a: Histogram for the month of April.

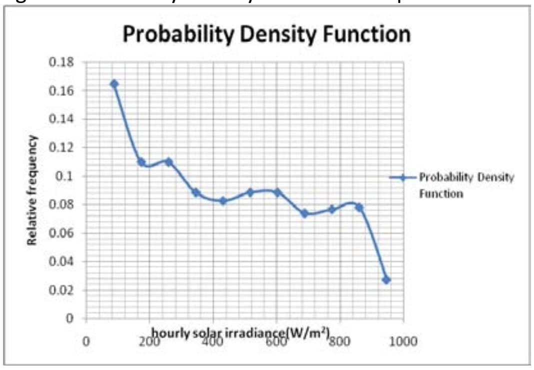


Fig 1.5: Probability density function for May

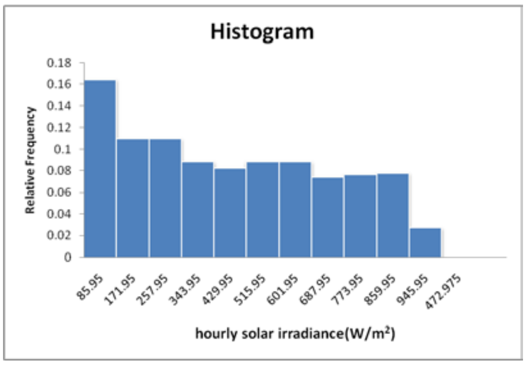


Fig 1.5a: Histogram for the month of May

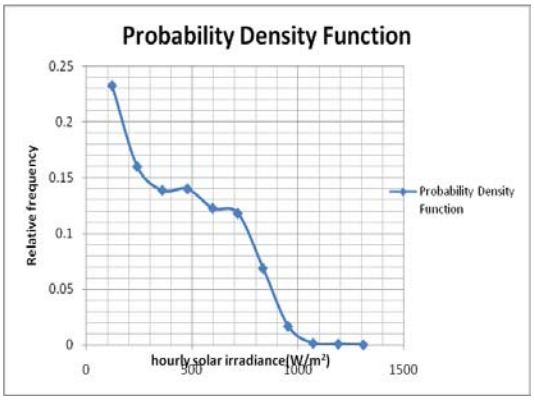


Fig 1.6: Probability density function for June

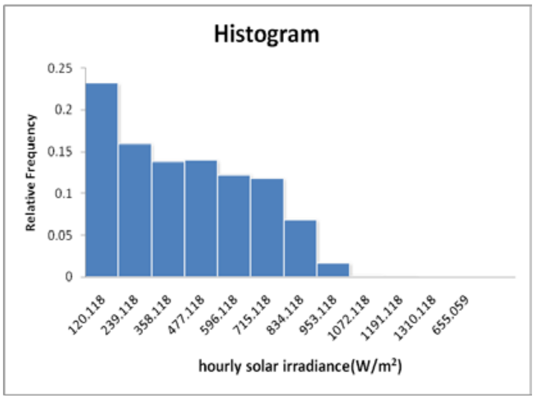


Fig 1.6a: Histogram for the month of June

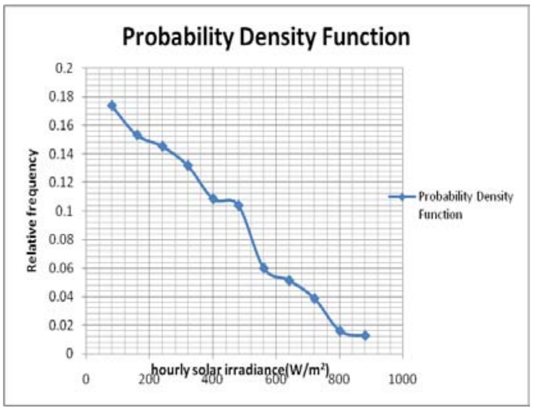


Fig 1.7: Probability density function for July

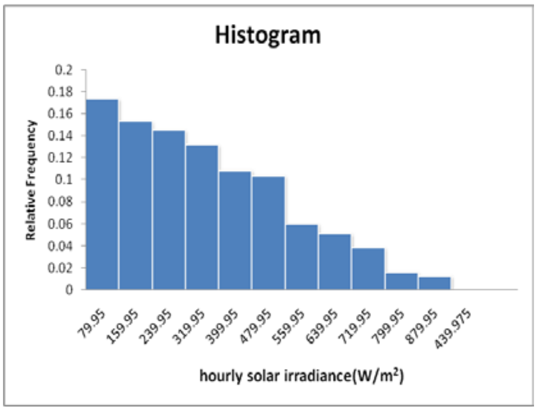


Fig 1.7a: Histogram for the month of July

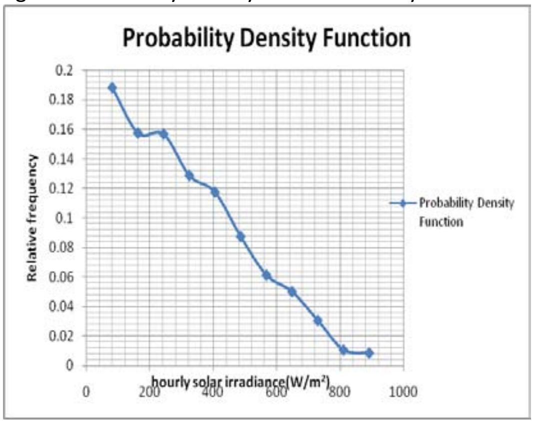


Fig 1.8: Probability density function for August

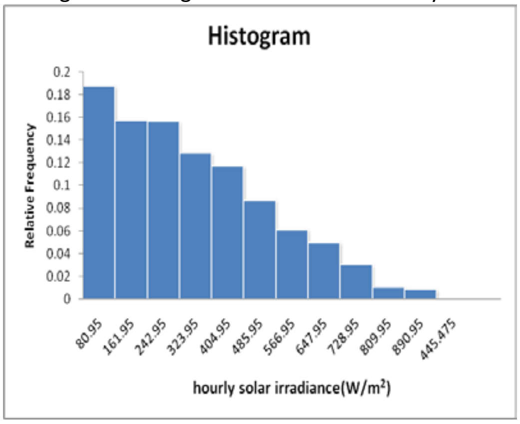


Fig 1.8a: Histogram for the month of August

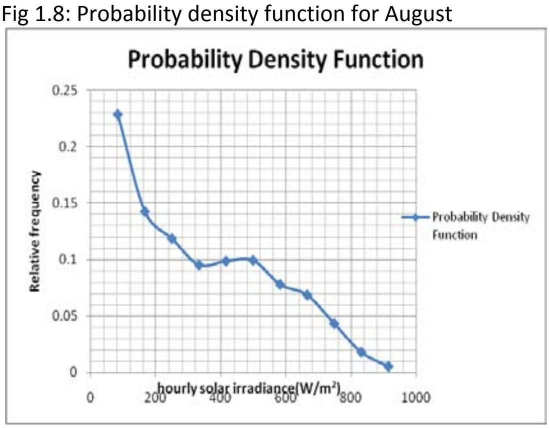


Fig 1.9: Probability density function for September

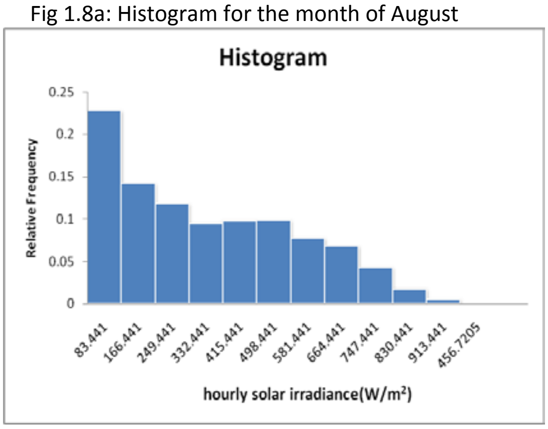
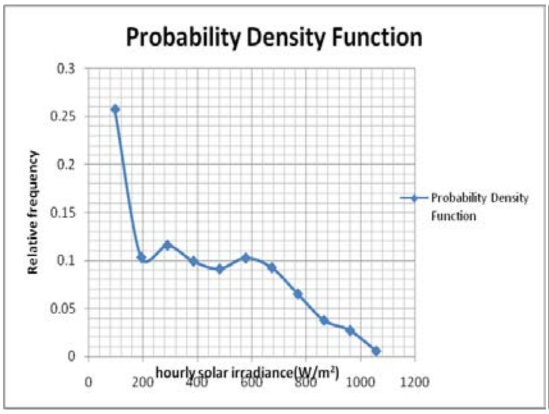


Fig 1.9a: Histogram for the month of September



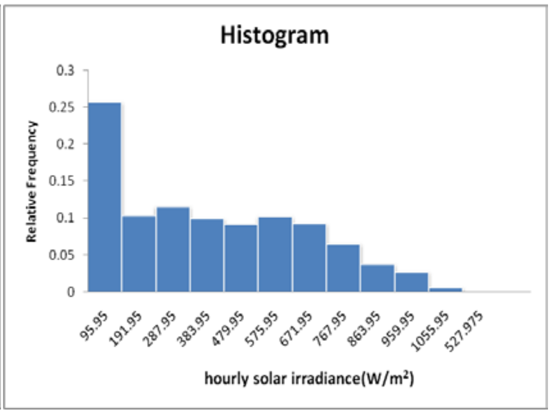
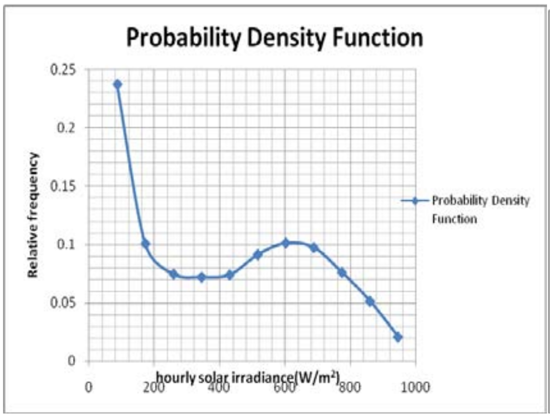


Fig 1.10: Probabilility density function for October

Fig 1.10a: Histogram for the month of October

Probability Density Function and Histogram for November

From a random sample of size one thousand five hundred selected from the population .The histogram and the probability density function for the month of November is presented in the Figure 1.11 and tables 1.11a below respectively.



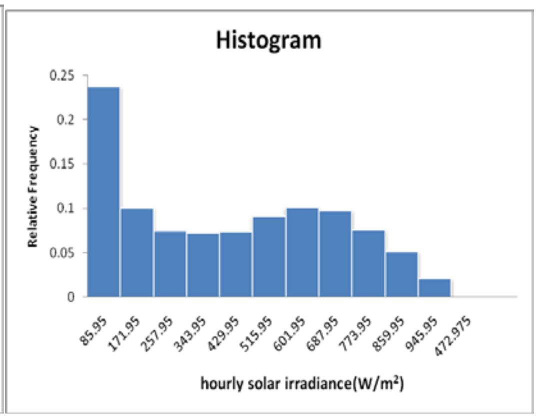
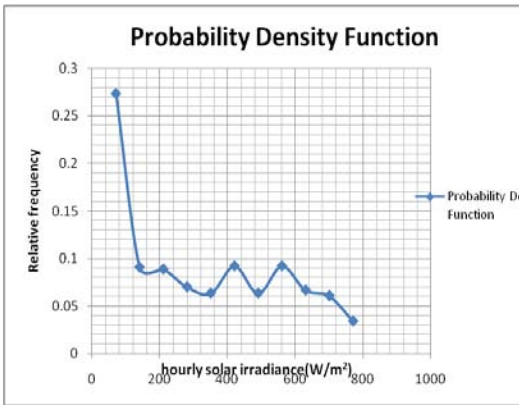


Fig 1.11: Probability density function for November

Fig 1.11a: Histogram for the month of November



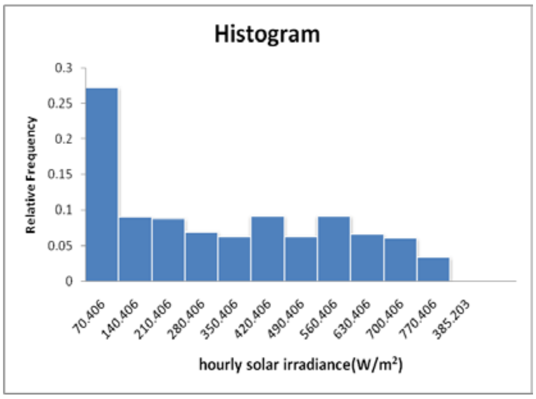


Fig 1.12: Probability density function for December

Fig 1.12a: Histogram for the month of December

Mean Square Errors of the Distributions

The standard distribution with the smallest mean square error with respect to the data was considered the best candidate distribution for the particular month under consideration.

Due to the large number of distributions dealt with, the remaining tables and charts have been omitted here and rather presented in the appendix.

Tables 2.1 and 2.2 below show the mean square errors for various standard probability distributions for the months of January and February respectively. We find that the lognormal distribution is the best fitted distribution for the month of January since it has the smallest mean square error while in the month of February the following class of probability distributions all fit best: Exponential, Weibull, Gamma, Lognormal, and Geometric distributions.

Mean Square Errors of the Distributions for Jan and Feb

Table 2.1: Distribution Mean Square Error for Jan

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	0.000003
Weibull Distribution	0.000003
Gamma Distribution	0.000003
Lognormal Distribution	2.54456E-06
Beta Distribution	2.005E+191
Geometric Distribution	0.000003

Table 2.2: Distribution Mean Square Error for Feb

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	3.7037E-08
Weibull Distribution	3.7037E-08
Gamma Distribution	3.7037E-08
Lognormal Distribution	3.59166E-08
Beta Distribution	2.88371E+34
Geometric Distribution	3.7037E-08

Mean Square Errors of the Distributions for March and May

Tables 2.3 and 2.4 below show the mean square errors for various standard probability distributions for March and May respectively. The log normal distribution can be seen to be the best fitted distribution for the month of March and May since it has the smallest mean square error in both cases.

Table 2.3: Distribution Mean Square Error for March

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	0.000003
Weibull Distribution	0.000003
Gamma Distribution	0.000003
Lognormal Distribution	5.61688E-06
Beta Distribution	1.79165E+60
Geometric Distribution	0.000003

Table 2.4: Distribution Mean Square Error for May

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	2.5037E-05
Weibull Distribution	2.5037E-05
Gamma Distribution	2.5037E-05
Lognormal Distribution	4.98008E-07
Beta Distribution	1.29291E+81
Geometric Distribution	2.5037E-05

Mean Square Errors of the Distributions for April

The table 2.5 below shows the mean square errors for the various standard probability distributions under consideration for the month of April. The Weibull, exponential, gamma and the geometric distribution as can be seen below is the best fitted distribution for the month of April since it has the smallest mean square error.

Table 2.5: Distribution Mean Square Error for April

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	3.7037E-08
Weibull Distribution	3.7037E-08
Gamma Distribution	3.7037E-08
Lognormal Distribution	7.10438E-06
Beta Distribution	4.0449E+248
Geometric Distribution	3.7037E-08

Table 2.8: Distribution Mean Square Error for July

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	1.33333E-06
Weibull Distribution	1.33333E-06
Gamma Distribution	1.33333E-06
Lognormal Distribution	7.5403E-06
Beta Distribution	1.2338E+233
Geometric Distribution	1.33333E-06

Mean Square Errors of the Distributions for June and Sept

Tables 2.6 and 2.7 below show the mean square errors for the various standard probability distributions under consideration for June and September respectively. The Exponential, Gamma, Weibull and Geometric distributions as can be seen below were the best fitted distributions for both the months of June and September since they all have the smallest mean square error

Table 2.6: Distribution Mean Square Error for June

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	3.7037E-08
Weibull Distribution	3.7037E-08
Gamma Distribution	3.7037E-08
Lognormal Distribution	6.79067E-06
Beta Distribution	1.0862E+237
Geometric Distribution	3.7037E-08

Table 2.7: Distribution Mean Square Error for Sept

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	9.25926E-07
Weibull Distribution	9.25926E-07
Gamma Distribution	9.25926E-07
Lognormal Distribution	7.11702E-06
Beta Distribution	8.0732E+235
Geometric Distribution	9.25926E-07

Mean Square Errors of the distributions for July

Table 2.8 below shows the mean square errors for the various standard probability distributions under consideration for July. The Exponential, Gamma, Weibull and Geometric distributions were the best fitted distributions for both the months of June and September since they each have the smallest mean square error.

Mean Square Errors of the Distributions for August

Table 2.9 below shows the mean square errors for the various probability distributions under consideration for August best fitted distribution for the month of August since it has the smallest mean square error.

Table 2.9: Distribution Mean Square Error for August

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	9.25926E-07
Weibull Distribution	9.25926E-07
Gamma Distribution	9.25926E-07
Lognormal Distribution	7.49618E-06
Beta Distribution	5.7038E+234
Geometric Distribution	9.25926E-07

Table 2.10: Distribution Mean Square Error for October

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	3.7037E-08
Weibull Distribution	5.7201E-128
Gamma Distribution	3.7037E-08
Lognormal Distribution	6.37252E-07
Beta Distribution	1.13913E+81
Geometric Distribution	3.7037E-08

Mean Square Errors of the Distributions for Oct and Nov

Tables 2.10 and 2.11 below show the mean square errors for various standard probability distributions for October and November respectively. The Exponential, Gamma, Weibull and Geometric distributions were the best fitted distributions for both the months of October and November since they all have the smallest mean square error.

Table2.11: Distribution Mean Square Error for Nov

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	3.33333E-07
Weibull Distribution	3.33333E-07
Gamma Distribution	3.33333E-07
Lognormal Distribution	7.33666E-06
Beta Distribution	1.8199E+191
Geometric Distribution	3.33333E-07

Table 2.12: Distribution Mean Square Error for December

DISTRIBUTION	MEAN SQUARE ERROR
Exponential Distribution	2.37037E-06
Weibull Distribution	2.37037E-06
Gamma Distribution	2.37037E-06
Lognormal Distribution	4.14557E-06
Beta Distribution	1.0932E+178
Geometric Distribution	2.37037E-06

Mean Square Errors of the Distributions for December

Table 2.12 below shows the mean square errors for the various standard probability distributions under consideration for December. The Exponential, Gamma, Weibull Lognormal and Geometric distributions were the best fitted distributions for the month of December since they all have the smallest mean square error.

Monthly Probability Distributions

The randomly selected solar irradiation data was fitted into the various probability distributions and the resulting graphs of their probability distributions are presented for the various months of January march and May. As discussed in section 2.3 the lognormal probability distribution can best fit the month of January march and May. Fig 2.16 below gives the graph of the lognormal distribution for the month of January march and May solar irradiation.

As discussed in section in section 2.3 the Exponential, Gamma, Weibull Lognormal and Geometric distributions can best fit the month of April June September October and November. Fig 2.16 to fig 2.21 below gives the graph of the above mentioned probability distribution for the month of April, June, September, October and November

As discussed in section in section 2.3 the Exponential, Gamma, Weibull Lognormal and Geometric distributions can best fit the month of February, July and December. Fig 2.21 to fig 2.25 below gives the graph of the above mentioned probability distribution for the month of February, July and December

CONCLUSION

It is well known that the solar irradiation reaching the surface of the earth is not normally distributed. There is the need to find the probability density function that best describes the sunshine hours in the various month of the year. In the present study, the method of mean square error was used to obtain the best distribution among a selection of standard distributions for each month of the year. In each case the standard probability distribution with the smallest mean square error relative to observed data was considered the best probability density function.

It was thus found that the solar irradiation for the month of April was Weibull distribution January, March and May is Lognormal distributed, and those for the months of June, August, September October and November follows Exponential, Weibull, Geometric and gamma distribution while February July and December were Lognormal, Exponential, Weibull, Geometric and gamma distributed. The hourly solar irradiation data for Kumasi, Ghana, was analyzed in order to determine standard probability distribution models pertaining to the various months of the year, as well as for clusters of months of the year with similar patterns. From the analysis conducted it was also found that hourly solar irradiation for January, March and May could be fitted to the lognormal probability distribution while the month of April could be equally well fitted to the Weibull, exponential, gamma and the geometric distributions. The months of June, July, August, September, October, November and December could be equally well be fitted to the Exponential, Weibull, Geometric and Gamma distributions.

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