School Bus Routing: A Case Study of Wood Bridge School Complex, Sekondi-Takoradi, Ghana

John AwuahAddor[1], Samuel Kwame Amponsah[2], Jonathan Annan[3], Charles Sebil[4]

ARTICLE INFO

Available Online December 2013

Key words:
Ant Colony;
Integer Programming;
Metaheuristics;
Vehicle Routing Problem;
Travelling Salesman Problem.

ABSTRACT

This research article presents a School Bus Routing Problem of Wood Bridge School Complex, Sekondi-Takoradi, Ghana. The problem was formulated as an Integer Programming Model and an Ant Colony Based Meta-heuristic for the Travelling Salesman Problem was used to solve the problem. Data on distances were collected and coded using Matlab. Our proposed model revealed a tremendous improvement in the total route length by approximately 32%.

Introduction

In advanced countries, most of the School Bus Transportation Services (SBTS) are run by public transports (Bowerman, Hall and Calamai, 1995). In Ghana, school transportation services, in most private schools, are executed using school buses. The task of finding an efficient route is an important logistic problem (Bell and McMullen, 2004).This research article seeks to analyze the efficiency of school bus routing of Woodbridge School Complex. Savas (1978) outlined efficiency, effectiveness and equity as performance criteria for the provision of school bus routing. According to the author, efficiency measures the ratio of the level of service to the cost required to provide the service. The level of service required in providing an efficient school bus routing is fixed for a particular situation, hence the main variable in determining the efficiency of a particular solution is the total cost of providing the service in currency units or manpower. The efficiency of a solution can be measured by its cost (Bowerman, Hall and Calamai, 1995). In this instance, the problem involves finding the minimum cost of the combined routes for a number of vehicles in order to connect students from the school to a number of locations. The cost is directly related to the distance covered.

We shall formulate the school bus routing as single objective problem and application centered on the one developed by Schittekat, Sevauz and Sörensen(2009).We shall use the Ant Colony(a heuristic) to solve the school bus routing of Wood Bridge School Complex by taking in to consideration the actual distances between various picking points of the buses.

Related Works

The decision making process of ants are embedded in the artificial intelligence (AI) algorithm of a group of virtual ants which are used to provide solution to the Vehicle Routing Problem (Bell and McMullen, 2004).

Bell and McMullen (2004) established that the performance of ACO is competitive with other techniques used to generate solutions to the Vehicle Routing Problem.

Dorigo and Gambardella (1997) concluded that ACO out-performs other nature-inspired algorithms such as simulated annealing and evolutionally computation.

Abounacer et al., (2009) came out with the conclusion that ACO performs better than Genetic Algorithm (GA) in terms of cost calculations. However, the performances of both techniques in determining the number of vehicles, required for a given set T of students, are the same.

[1] Lecturer, Department of Mathematics and Statistics, Takoradi Polytechnic, Takoradi
[2] Lecturer, Department of Mathematics, Kwame Nkrumah University of Science and Technology,
[3] Lecturer, Department of Information Systems and Decision Science, School of Business
Kwame Nkrumah University of Science & Technology Kumasi
[4] Lecturer, Department of Mathematics, Kwame Nkrumah University of Science and Technology,

The Proposed Model

Assumptions
The assumptions of the proposed model are:
(i) Service is available to only students whose residences are not within walking distances from the school.
(ii) All students to be serviced must walk to an allowed bus stop.
(iii) A bus must visit a given stop only once.
(iv) Capacities of buses must not be exceeded.

Let,
K^{b} = Capacity of bus b
B = Number of buses
C_{ij} = Cost of traversing arc(i, j)
S = Set of all potential stops
T_{ti} = Binary variable that shows if a student t can walk to stop i or not
A = Set of all arcs between stops.
O = The school

Decision Variables
$N^{(b)}{}_{ij}$ = Number of times bus b traverses arc(i, j)

$$V^{(b)}{}_i = \begin{cases} 1, & \text{if bus } b \text{ visit stop } i \\ 0, & \text{otherwise} \end{cases}$$

$$P^{(b)}{}_{it} = \begin{cases} 1, if\ bus\ b\ picks\ up\ a\ student\ t\ at\ i \\ 0, otherwise \end{cases}$$

The problem is:

$$minimize\ \ f = \sum_{i \in s} \sum_{j \in s} C_{ij} \sum_{b=1}^{B} N^{(b)}{}_{ij} \quad (1)$$

$$s.t.\quad \sum_{b=1}^{B} V_0{}^{(b)} \le B\,, \quad b = 1, \dots, B \qquad (2)$$

$$\sum_{j \in s} N^{(b)}{}_{ij} = \sum_{j \in s} N^{(b)}{}_{ji} = V^{(b)}{}_i\ ,\ \ \forall\ i \in S, b = 1, \dots, B \qquad (3)$$

$$\sum_{i \in T} \sum_{j \notin T} N^{(b)}{}_{ji} \ge V^{(b)}{}_h, \quad \forall T \le S \backslash \{0\}, h \in t,\ \ b = 1, \dots, B \qquad (4)$$

$$\sum_{b=1}^{B} V^{(b)}{}_i \ \le 1,\ \ \forall i \in S \backslash \{0\} \qquad (5)$$

$$\sum_{b=1}^{B} P^{(b)}{}_{it} \ \le T_{ti}\ , \forall t\ \in T, i \in S \qquad (6)$$

$$\sum_{i \in S} \sum_{t \in T} P^{(b)}{}_{it} \le K^{(b)},\ \ b = 1, \dots, B \qquad (7)$$

$$P^{(b)}{}_{it} \le V^{(b)}{}_i, \forall i \in S, t \in T, b = 1, \dots, B \qquad (8)$$

$$\sum_{i \in s} \sum_{b=1}^{B} P^{(b)}{}_{it} = 1\ ,\ \ \forall t \in T \qquad (9)$$

$$V_i{}^{(b)} \in \{0, 1\}\,, \forall i \in S\ ,\ \ b = 1, \dots, B \qquad (10)$$

$$N_{ij}{}^{(b)} \in \{0, 1\}, \forall i, j \in S \backslash i \ne j \qquad (11)$$

$$P_{it}{}^{(b)} \in \{0,1\}, \forall i, j \in S \backslash i \ne j \qquad (12)$$

The objective function (1) minimizes the total length of all routes covered by the buses. Constraint (2) guarantees that all buses start from the school. Constraint (3) guarantees that if bus b visits stop i then one arc is traversed by b entering and exiting i. Constrain(4) prevents the formation of sub-tours. This means that each cut defined by a customer set T is crossed by a number of arcs not less than the minimum number of buses $n(B)$ required to serve set T. Constraint (5) guarantees that a bus visits a particular stop not more than one. Constraint (6) ensures that every student walks to his single designated stop only. Constraint (7) guarantees that respective capacities of buses are not exceeded. Constraint (8) guarantees that a student t designated to stop i is picked up by bus b provided b visits stop i. Constraint (9) ensures that all students are picked up only once. Finally,(10),(11) and (12) represent the binary integrality constraints on all decision all decision variables.

Ant Colony Optimization

Ant Colony Optimization (ACO) is part of a larger field of swarm intelligence in which the behavior patterns of bees, termites, ants and other social insects are studied in order to simulate processes. The ability of insect swarms to thrive in nature and solve complex survival tasks appeals to scientists developing computer algorithms needed to solve similarly complex problems. Artificial intelligence algorithms such as ant colony optimization are applied to large combinatorial optimization problems and are used to create self-organizing methods for such problems. Ant colony optimization is a meta-heuristic technique that uses artificial ants to find solutions to combinatorial optimization problems. ACO is based on the behavior of real ants and possesses enhanced abilities such as memory of past actions and knowledge about the distance to other locations. In nature, an individual ant is unable to communicate or effectively hunt for food, but as a group, ants possess the ability to solve complex problems and successfully find and collect food for their colony. Ants communicate using a chemical substance called pheromone (Dorigo and Gamberdella, 1997).

As an ant travels, it deposits a constant amount of pheromone that other ants can follow. Each ant moves in a somewhat random fashion, but when an ant encounters a pheromone trail, it must decide whether to follow it. If it follows the trail, the ant's own pheromone reinforces the existing trail, and the increase in pheromone increases the probability of the next ant selecting the path. The more ants travel on a path, the more attractive the path becomes for subsequent ants. An ant using a short route to a food source will return to its nest sooner and mark its path twice before other ants return. This directly influences the selection probability for the next ant leaving the nest. Over time, as more ants are able to complete the shorter route, pheromone accumulates faster on shorter paths and longer paths are less reinforced.

The evaporation of pheromone also makes less desirable routes more difficult to detect and further decreases their use. The continued random selection of paths by individual ants helps the colony discover alternate routes and insures successful navigation around obstacles that interrupt a route. Trail selection by ants is a pseudo-random proportional process and is a key element of the simulation algorithm of ant colony optimization (Dorigo and Gamberdella, 1997).

ACO was first applied to the travelling salesman problem and the quadratic assignment problem (Dorigo, 1992). Ever since, it has been applied to other problems, which include but not limited to the space planning problem (Bland, 1999), the machine tool tardiness problem (Bauer, Bullnbeimer and Hartl, 1999) and the multiple objective Just In Time (JIT) sequencing problem (McMullen, 2001).

Route construction

Using ACO, an individual ant simulates a vehicle, and its route is constructed by incrementally selecting customers until all customers have been visited. Initially, each ant starts at the depot and the set of customers included in its tour is empty. The ant selects the next customer to visit from the list of feasible locations and the storage capacity of the vehicle is updated before another customer is selected. The ant returns to the depot when the capacity constraint of the vehicle is met or when all customers are visited. The total distance L is computed as the objective function value for the complete route of the artificial ant. The ACO algorithm constructs a complete tour for the first ant prior to the second ant starting its tour. This continues until a predetermined number of ants m each constructs a feasible route.

Using ACO, each ant must construct a vehicle route that visits each customer. To select the next customer j, the ant uses the following probabilistic formula (Dorigo and Gamberdella, 1997)

$$j = \begin{cases} argmax\{(T_{iu})(\eta_{iu})^{\beta}\}, u \notin M_k, q \leq q_0 \\ S, otherwise \end{cases} \quad (13)$$

WhereT_{iu} is equal to the amount of pheromone on the pathbetween the current location i and possible locations u. Thevalue η_{iu} is defined as the inverse of the distance between the two customer locations and the parameter β establishesthe importance of distance in comparison to pheromonequantity in the selection algorithm ($\beta > 0$). Locationsalready visited by an ant are stored in the ants workingmemory M_k and are not considered for selection. The value$q$ is a random uniform variable [0,1] and the value q_0 is aparameter. When each selection decision is made, the antselects the arc/edge with the highest value from (13) unless q isgreater than q_0. In this case, the ant selects a randomvariable (S) to be the next customer to visit based on theprobability distribution of P_{ij}, which favors short paths withhigh levels of pheromone:

$$P_{ij} = \begin{cases} \dfrac{(T_{iu})(\eta_{iu})^{\beta}}{\sum_{u \notin M_k}(T_{iu})(\eta_{iu})^{\beta}} \text{if } j \notin M_k \\ 0 \; otherwise \end{cases} \quad (14)$$

Using (13) and (14), each ant may either follow the most favorable path already established or may randomly select a path to follow based on a probability distribution established by distance and pheromone accumulation. If the vehicle capacity constraint is met, the ant will return to the depot before selecting the next customer. This selection process continues until each customer is visited and the tour is complete.

Trail Updating

To improve future solutions, the pheromone trails of the ants must be updated to reflect the ant's performance and the quality of the solutions found. This updating is a key element to the adaptive learning technique of ACO and helps to ensure improvement of subsequent solutions (Bell and McMullen, 2004). Trail updating includes local updating of trails after individual solutions have been generated and global updating of the best solution route after a predetermined number of solutions m has been accomplished.

First, local updating is conducted by reducing the amount of pheromone on all visited arcs in order to simulate the natural evaporation of pheromone and to ensure that no one path becomes too dominant. This is done with the following local trail updating equation:

$$T_{ij} = (1 - \alpha)\, T_{ij} + (\alpha)\, T_0 \quad (15)$$

Whereα is a parameter that controls the speed ofevaporation and T_0 is equal to an initial pheromone value assigned to all arcs in graph G. For this study, T_0 is equal tothe inverse of the best known route distances found for theparticular problem.After a predetermined number of ants, m construct a feasible route, global trail updating is performed by adding pheromone to all of the arcs included in the best route found by one of the m ants. Global trail updating is accomplished according to the equation:

$$T_{ij} = (1 - \alpha)\, T_{ij} + \alpha L^{-1} \quad (16)$$

This updating encourages the use of shorter routes and increases the probability that future routes will use the arcs/edges contained in the best solutions. This process is repeated for a predetermined number, k of iterations and the best solutionfrom all of the iterations is presented as an output of themodel and should represent a good approximation of theoptimal solution for the problem.

Methodology

The data involves measurement of distances from one bus stop to the other. Distances were recorded using a car, which records distances (in kilometers) digitally. This was later converted to x and y coordinates, which gave rise to the Distance Matrix Table. The results were obtained with the aid of a programme written in Matlab, which was run on a Toshiba Laptop Computer with hard disc drive of 73MB

Table 1: Distance Matrix Table

S/N		BUS 1			BUS 2			BUS 3			BUS 4	
		X	***Y***		***X***	***Y***		***X***	***Y***		***X***	***Y***
1		4.6	0.2		4.6	1.4		4.5	-1.6		0.8	0.6
2		-1.6	0.4		-1.3	0.6		-0.5	0.2		0.2	-0.3
3		-0.9	-0.3		1.9	1.0		-1.0	0.1		0.1	-0.2
4		-0.6	0.7		0.6	0.5		-0.2	0.4		-0.4	1.2
5		-0.1	0.1		1.5	-0.1		1.1	-2.6		1.4	1.1
6		0.1	0.0		1.3	0.4		0.2	0.4		-0.5	0.2
7		1.4	-1.9		0.2	0.1		1.3	0.2		0.3	-0.1
8		-0.3	-0.3		-2.7	0.6		1.5	-0.3		1.4	-3.3
9		1.2	-0.1		2.2	2.8		1.2	-3.1		-1.2	-1.0
10		0.4	0.0		0.97	0.5		-3.7	1.1		0.1	-0.6
11		0.6	-0.2		-1.2	2.4		0.5	0.3		-0.5	-1.2
12		0.8	1.4		-1.0	0.2		-1.9	0.8		0.2	-0.1
13		2.5	1.0		-2.0	1.3		0.2	-0.2		-1.8	0.1
14		0.3	0.6		-0.3	0.3		0.3	1.6		1.2	-3.1
15		0.4	1.0		-0.2	-0.1		3.8	2.6			
16		-1.7	0.1		0.7	0.6		0.3	-0.7			
17		1.2	-3.1		1.1	-0.2		-1.6	0.9			
18					0.5	0.2		-0.4	0.5			
19					0.2	-0.1		-1.6	0.9			
20					-0.1	1.4		-0.1	0.1			
21					1.2	3.1		-0.4	0.5			

Results

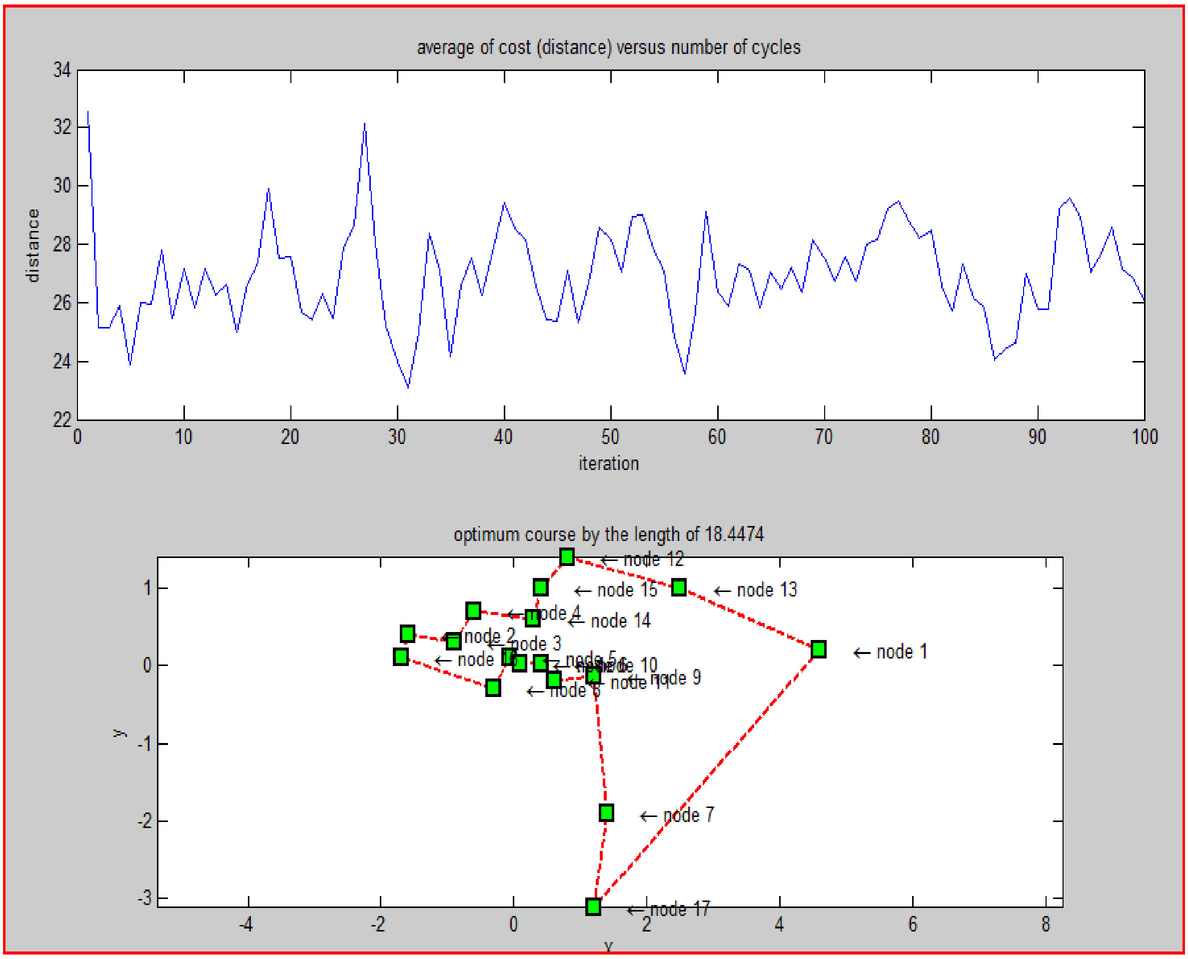

Figure 1: Ant colony result of 200 ants for bus 1.

Figure 1 shows that for 200 ants the best ant will cover an optimum course by a distance of about 18km, which is the same as that which was covered by 100 ants (see Table 2). This means that the optimal route length covered by bus 1 using ant colony is approximately 18km, and is given by the optimal tour:
1→17→7→9→11→10→6→5→8→16→2→3→4→14→15→12→13→1

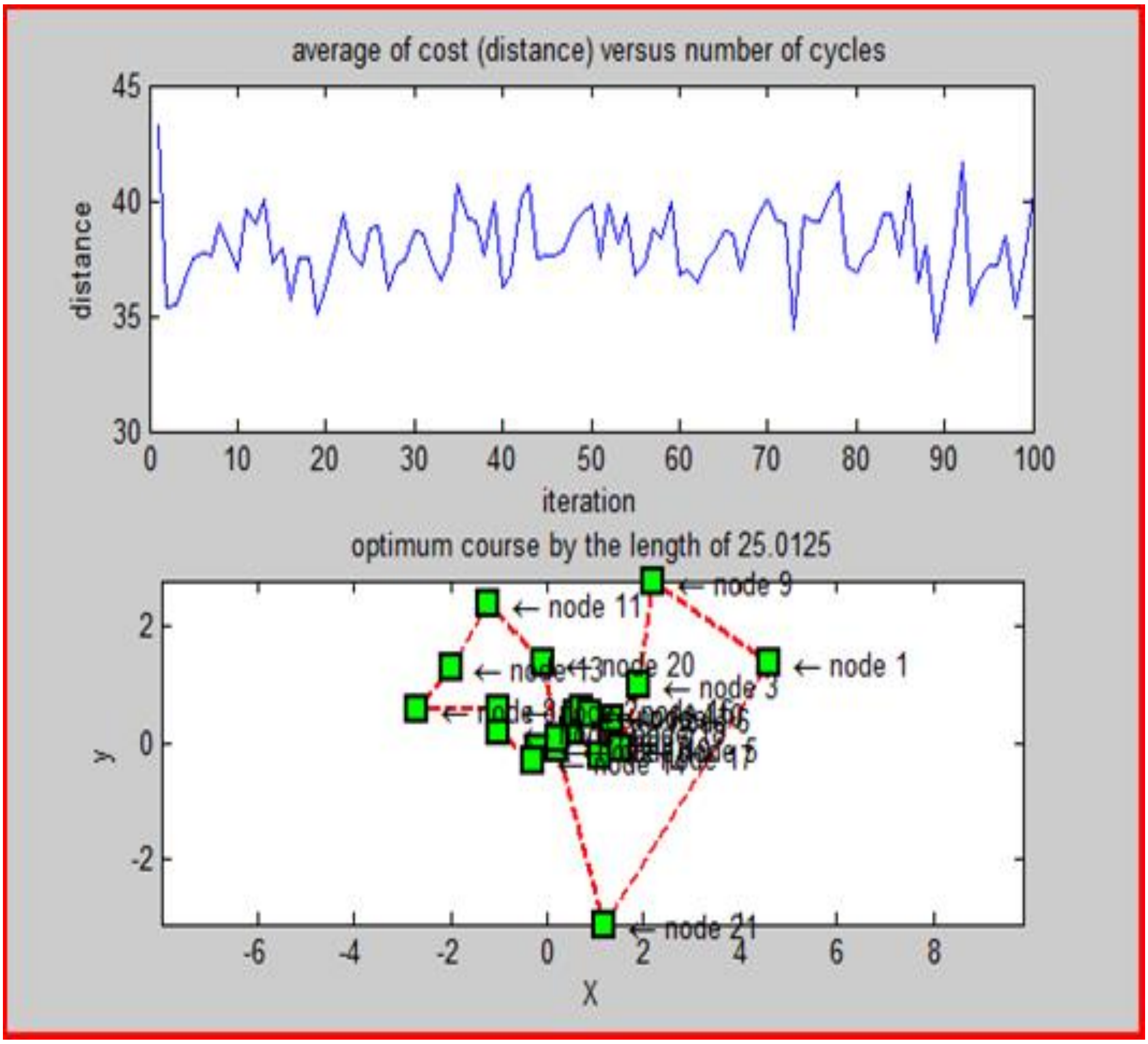

Figure 2: Ant colony result of200 ants for bus 2.

Similarly, in Figure 2, the best ant out of 200 will cover an optimum course by the length of approximately 25km. This is equivalent to that which was covered by 100 ants (see Table 2). Thus, the optimal course of bus 2 via ant colony optimization is about 25km. The optimal route and order of movement (clockwise) from the source node (1) is displayed in the lower panel.

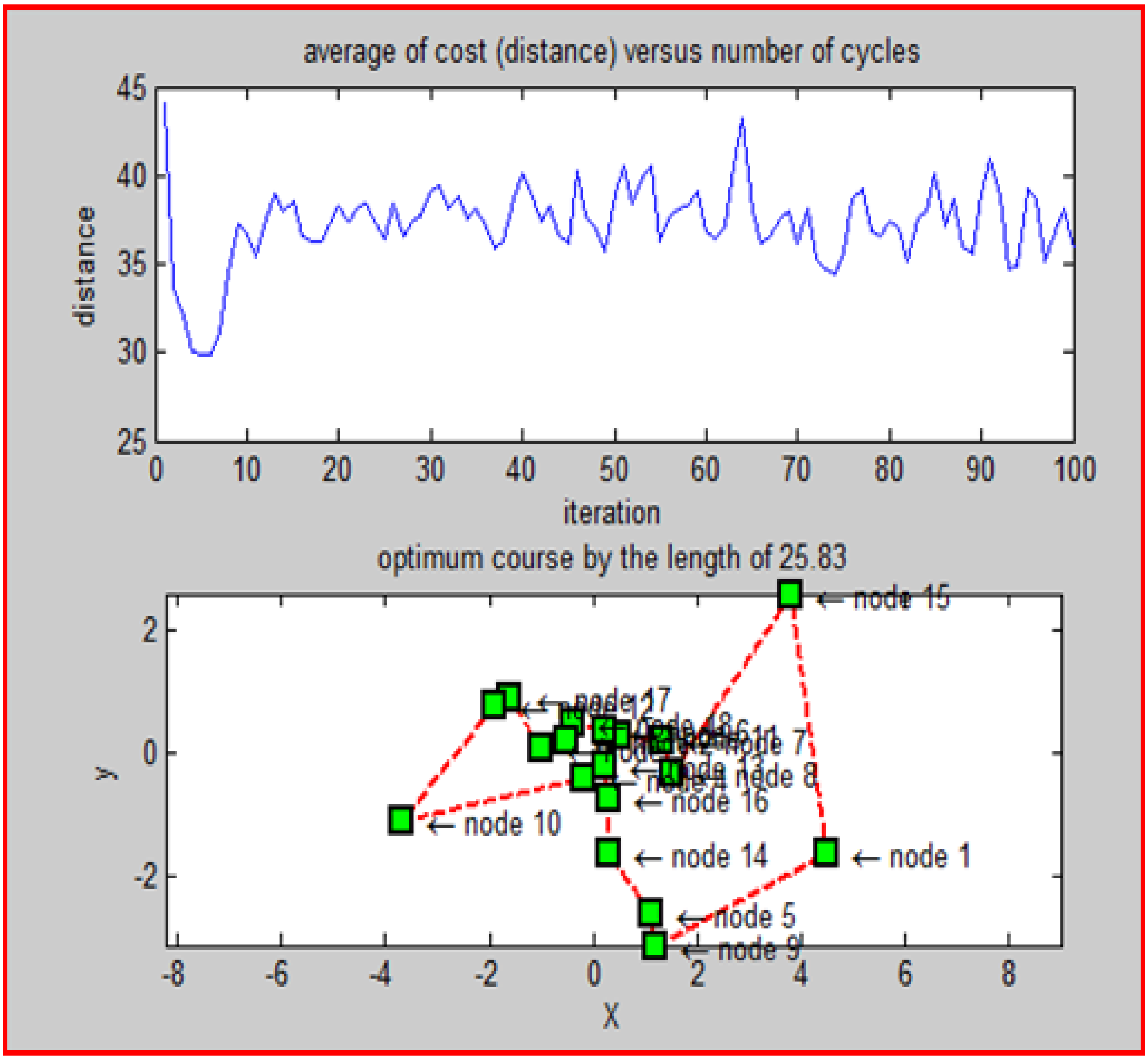

Figure 3: Ant colony result of 200 ants for bus 3.

Figure 3depicts that for 200 ants; the optimum length completed by the best out of 200 ants is approximately 26km. The optimal course from the source node is given by
1→9→5→14→16→13→4→10→12→17→2→3→18→6→11→7→8→15→1

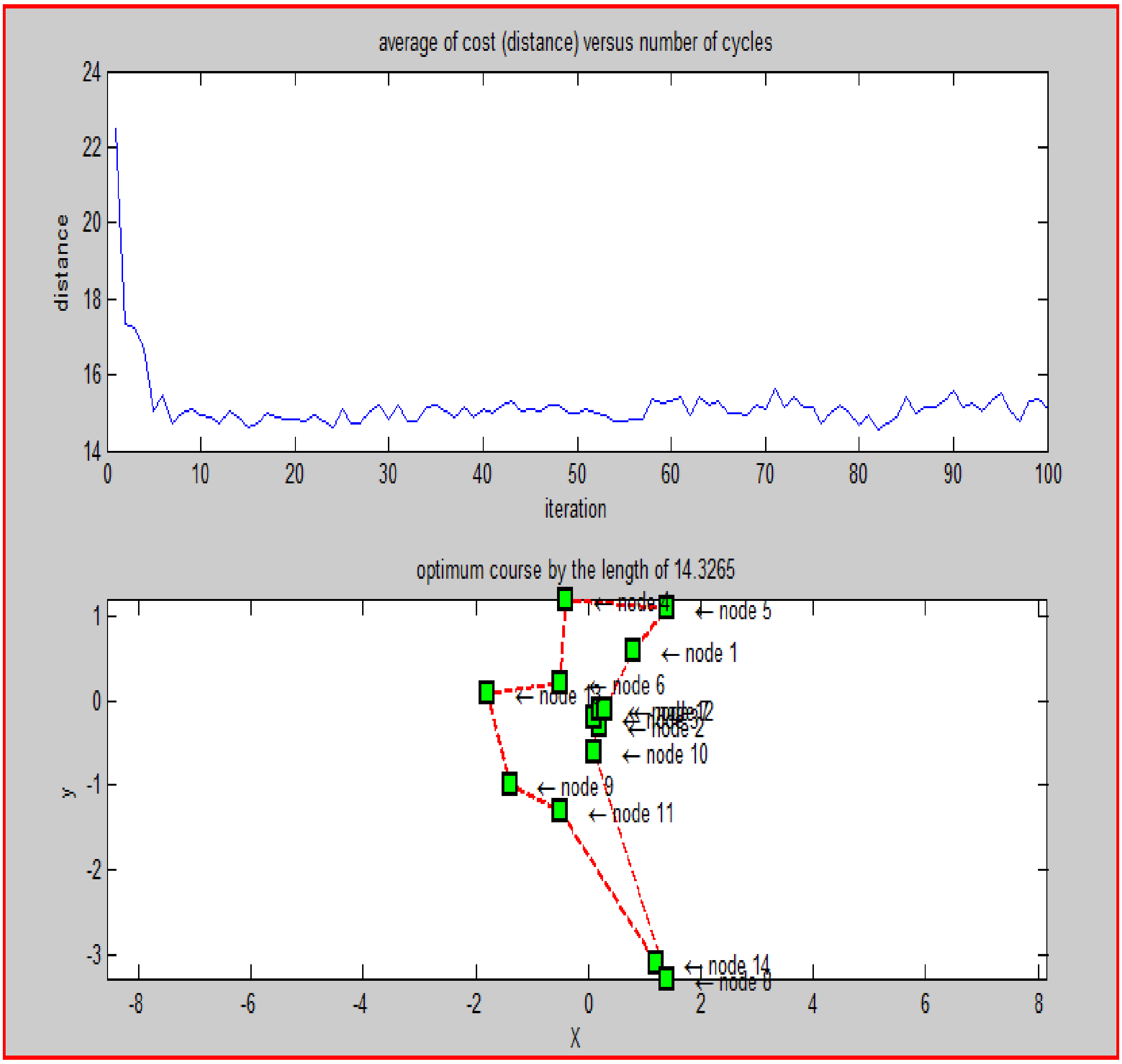

Figure 4: Ant colony result of 100 ants for bus 4.

From Figure 4, the best out of 100 ants records an optimal route length of approximately14km, representing about the same optimal distance covered by the best out of 50 ants (see Table 2). It can therefore be summarized that the optimum course for bus 4 using ant colonies is by the length of approximately 14km. The optimal route from the source node is given by 1→12 →7 → 3→2 10→8→14→11→ 9→13→6→4→5→1.

Table 2: Summary of Ant Colony Results

BUS	ORIGINAL LENGTH (KM)	NUMBER OF ANTS	OPTIMAL LENGTH (KM)	PERCENTAGE CHANGE
1	24.4	10	25.4937	4.4824
		15	24.4127	-0.0520
		50	20.7094	-15.1254
		100	18.4474	-24.3959
		200	18.4474	-24.3959
2	40.7	8	39.1916	-3.7061
		50	25.2950	-37.8501
		100	25.1028	-38.3223
		200	25.0125	-38.5442
3	39.94	7	40.0055	0.1640
		10	37.7670	-5.4410
		50	29.5978	-25.8943
		100	25.8300	-35.3280
		200	26.117	-35.3280
4	17.8	5	20.5619	15.5163
		15	16.6908	-6.2315
		50	14.3265	-19.5140
		100	14.3265	-19.5140

Source: Results of author's field survey. May, 2011

Table 1 presents a summary of the ant colony results for buses 1, 2, 3 and 4. The original lengths of tour by the four buses are shown in column 2, whilst the optimal route lengths obtained from ant colony results for the various ants numbers are displayed in column 4. The last column shows the percentage change in the original route lengths. The negative changes depict improvement in optimal solution whilst the positive changes shows otherwise.

Discussions

The result in Figure 1 shows the optimal route length that will yield the optimal (reduced) cost for bus 1. From Figure 1 and Table 2, ants' numbers corresponding to 10 will be inefficient since it increases the original tour length for bus 1 by approximately 4.5%.If 15 ants are considered, the optimal cost will be given by a route length of about 24kilometres. This shows a result close to the actual distance covered by bus 1, which could mean that the school operates based on a route constructed by about 15 ants. However, for ant numbers 50, 100 and 200 the optimal course for bus 1 is by the length of approximately 20km, 18km and 18km, respectively. It can therefore be concluded that the optimal course for bus 1 is by the length of about 18km, which represent a reduction in cost by about 25%(1/4).

The experimented result in Figure 2 shows the optimal course for the tour of bus 2. 8 ants will yield an optimal route length of about 39km, which is somehow close to the original tour length covered by bus 2. For 50, 100 and 200 ants the route length will be reduced by about 38%, 38% and 39%, respectively (refer to Table 2). Hence, the optimal course for bus 2 is by length of close to 25 km.

The optimal course for bus 3 is depicted by the Figure 3.For 7 ants the optimal course is by the length of about 40km. Thus, the existing length of tour covered by the bus 3 depicts the results of 7 ants. This implies that for ants' number greater than 7, the optimal course taken by bus 3 will begin to improve. Ants' numbers 10 and 50 reduced the cost by close to 5% and 26% respectively. Slimily 100 and 200 ants reduce the route length for bus 3 by the same percentage of 35%.As a result, the optimal route length for the bus 3 is approximately 26km.

From Table 2, the tour constructed by 5 ants will be inefficient for consideration by bus 4 since it increases the original tour length constructed by bus 4 by approximately 16%. 15 ants construct an optimal course by length of about 17km, which also represents about 6%increase in cost. This suggests that the optimal route length will improve for number of ants greater than 15.For 50 and 100 ants the optimal route length converges to approximately 14km representing about 20% optimization of cost of transport service rendered by bus 4 (as depicted in Figure 4).

In general, the ant colony optimization has performed creditably well by reducing the original route length by about 40km, which is a reduction from 122.86km to 83.6164km and represents about 32% reduction in total cost (see Table 2).

It could also be discerned from the figures and table that the number of ants required for route length to begin to improve varies with the problem size. For example in Table 1, the route length will begin to improve for at least 16 ants as in the case of bus 1. Also for bus 2, at least 8 ants are required for route length to begin to improve. For buses 3 and 4 at least 8 ants and 15 are required. Since the number of nodes for the buses 1, 2, 3 and 4 respectively are 17, 21, 18 and 14, it is logic to contend that for smaller number of nodes, high number of ants are required for optimal route length to begin to improve than are required for large number of nodes.

Conclusions

The ant colony algorithm has proven itself to be a powerful tool for solving strong combinatorial optimization problems like the VRP. The results evince the possibility of the ant colony optimization heuristic to converge the solution to optimality. Based on the experimented results and the discussions above, the following conclusions can be made.

(i) Optimal route lengths for buses 1, 2, 3 and 4 are approximately 18km, 25km 26km and 14km respectively.
(ii) Cost of services rendered by buses 1, 2, 3 and 4 are reduced by 25%, 39%, 35% and 20%, respectively.
(iii) In general, the total cost of transportation is reduced by approximately 32% and the optimal routes are displayed in figures 1, 2, 3 and 4.
(iv) Number of ants required for the commencement of improvement in optimal route length varies inversely with the number of nodes.

REFERENCES

Abounacer R., Bencheikh, G., Boukachour J., Dkhissi B. andElhilali A.A. (2009). Population Metaheuristic to solve the Professional Staff Transportation Problem, International Journal of Computer Science and Network Security (IICSNS-06), 9(7):22-34.

Baver A., Bullnbeimer B., Hartl R. F. and Straues C. (1999). Ant Colony Optimization Approach for the Single Machine Tool Tardiness Problem.Proceedings of the 1999 Congress on Evolutionary Computation, p.1445-1450.

Bell J. E. and McMullen P.R. (2004).Ant Colony Optimization Techniques for the Vehicle Routing Problem. Advanced Engineering Informatics, 18:41-48.

Bland, J. A. (1999). Space Planning by Ant Colony Optimization. International Journal Computer Applied Technology, 6:320-328.

Bowerman R., Hall B., and Calamai P. (1995). A Multi-objective Optimization Approach to Urban School Bus Routing: Formulation and Solution Method. Transportation Research part A; Policy and Practice, 29A:107-123.

Dorigo M., Gamberdella L. M. (1997). Ant colony for the Travelling Salesman Problem. Biosystems, 43(1):73-81.

McMullen P. R. (2001). Ant colony Optimization Approach for addressing a JIT Sequencing Problem with Multiple Objectives. International Journal of Artificial Intelligence Engineering, 15:309-317.

Savas E. (1978).On equity in providing Public services. Journal of Management Science, 24:800-808.

Schittekat P., Sevaux M.,Sörensen K. (2009). A Mathematical Formulation for a School Bus Routing Problem, IEEE, p.1552-1556.